

Topic Outline for Calculus AB*

This outline of topics is intended to indicate the scope of the course, but it is not necessarily the order in which the topics are to be taught. Teachers may find that topics are best taught in different orders. (See the *Teacher's Guide – AP Calculus* for sample syllabi.) Although the examination is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits).

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior.

- Understanding asymptotes in terms of graphical behavior.
- Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)

Continuity as a property of functions.

- An intuitive understanding of continuity. (Close values of the domain lead to close values of the range.)
- Understanding continuity in terms of limits.
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

* There are no major changes to the Topic Outline from the May 2000, May 2001 edition of this Course Description. Minor editorial changes are not shaded.

II. Derivatives

Concept of the derivative.

- Derivative presented graphically, numerically, and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

Derivative at a point.

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function.

- Corresponding characteristics of graphs of f and f' .
- Relationship between the increasing and decreasing behavior of f and the sign of f' .
- The Mean Value Theorem and its geometric consequences.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives.

- Corresponding characteristics of the graphs of f , f' , and f'' .
- Relationship between the concavity of f and the sign of f'' .
- Points of inflection as places where concavity changes.

Applications of derivatives.

- Analysis of curves, including the notions of monotonicity and concavity.
- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.

Computation of derivatives.

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Basic rules for the derivative of sums, products, and quotients of functions.
- Chain rule and implicit differentiation.

III. Integrals

Interpretations and properties of definite integrals.

- Computation of Riemann sums using left, right, and midpoint evaluation points.
- Definite integral as a limit of Riemann sums over equal subdivisions.
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- Basic properties of definite integrals. (Examples include additivity and linearity.)

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line.

Fundamental Theorem of Calculus.

- Use of the Fundamental Theorem to evaluate definite integrals.
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation.

- Antiderivatives following directly from derivatives of basic functions.
- Antiderivatives by substitution of variables (including change of limits for definite integrals).

Applications of antidifferentiation.

- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling. In particular, studying the equation $y' = ky$ and exponential growth.

Numerical approximations to definite integrals. Use of Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

Change to Course Description for Calculus AB — Effective for the 2004 Examinations: Slope Fields

The following will be included in the topic outline for Calculus AB for the 2003-2004 academic year for the 2004 AP Examinations.

“Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.” [Applications of derivatives]

This topic has been part of the topic outline for Calculus BC since the 1998 AP Examinations.

Topic Outline for Calculus BC*

The topic outline for Calculus BC includes all Calculus AB topics. Additional topics are found in paragraphs that are marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fit immediately after their Calculus AB counterparts. Other topics may fit best after the completion of the Calculus AB topical outline. (See the *Teacher's Guide – AP Calculus* for sample syllabi.) Although the examination is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits).

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior.

- Understanding asymptotes in terms of graphical behavior.
- Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)

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Continuity as a property of functions.

- An intuitive understanding of continuity. (Close values of the domain lead to close values of the range.)
- Understanding continuity in terms of limits.
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

* **Parametric, polar, and vector functions.** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II. Derivatives

Concept of the derivative.

- Derivative presented graphically, numerically, and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

Derivative at a point.

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function.

- Corresponding characteristics of graphs of f and f' .
- Relationship between the increasing and decreasing behavior of f and the sign of f' .
- The Mean Value Theorem and its geometric consequences.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives.

- Corresponding characteristics of the graphs of f , f' , and f'' .
- Relationship between the concavity of f and the sign of f'' .
- Points of inflection as places where concavity changes.

Applications of derivatives.

- Analysis of curves, including the notions of monotonicity and concavity.
- + Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors.
- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- + Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.
- + Numerical solution of differential equations using Euler's method.
- + L'Hôpital's Rule, including its use in determining limits and convergence of improper integrals and series.

Computation of derivatives.

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Basic rules for the derivative of sums, products, and quotients of functions.
- Chain rule and implicit differentiation.
- + Derivatives of parametric, polar, and vector functions.

III. Integrals

Interpretations and properties of definite integrals.

- Computation of Riemann sums using left, right, and midpoint evaluation points.
- Definite integral as a limit of Riemann sums over equal subdivisions.
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- Basic properties of definite integrals. (Examples include additivity and linearity.)
- * **Applications of integrals.** Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and the length of a curve (including a curve given in parametric form).

Fundamental Theorem of Calculus.

- Use of the Fundamental Theorem to evaluate definite integrals.
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation.

- Antiderivatives following directly from derivatives of basic functions.
- + Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only).
- + Improper integrals (as limits of definite integrals).

Applications of antidifferentiation.

- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling. In particular, studying the equation $y' = ky$ and exponential growth.
- + Solving logistic differential equations and using them in modeling.

Numerical approximations to definite integrals. Use of Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

***IV. Polynomial Approximations and Series**

* **Concept of series.** A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence or divergence.

* **Series of constants.**

- + Motivating examples, including decimal expansion.
- + Geometric series with applications.
- + The harmonic series.
- + Alternating series with error bound.
- + Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p -series.
- + The ratio test for convergence and divergence.
- + Comparing series to test for convergence or divergence.

* **Taylor series.**

- + Taylor polynomial approximation with graphical demonstration of convergence. (For example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve.)
- + Maclaurin series and the general Taylor series centered at $x = a$.
- + Maclaurin series for the functions e^x , $\sin x$, $\cos x$, and $\frac{1}{1-x}$.
- + Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.
- + Functions defined by power series.
- + Radius and interval of convergence of power series.
- + Lagrange error bound for Taylor polynomials.